An Experimental Study of Nonlinear Standing Waves in Resonators with Numerical Comparison

Josh Finkbeiner, Xiaofan Li, Ganesh Raman, Chris Daniels, Bruce Steinetz

Lawrenson et. al. [Journal of the Acoustic Society of America, Nov. 1998] described the generation of shock-free high-amplitude pressure waves in closed cavities using large equipment and resonators to produce the reported effects. An attempt is made to generate shock-free high-amplitude pressure waves using relatively small resonators. Ambient air is used as the working fluid. A small cylindrical resonator is tested resulting in the lack of a shocked waveform while a larger model of the same shape produces shock waves. A small conical resonator produces shock-free pressure waves at resonance, but the amplitude of these waves is small. A larger cone resonator model produces shock-free pressure waves of higher amplitude. A large horn-cone resonator also produces shock-free high amplitude pressure waves. A numerical model is used to compare the experimental results to theoretical results. The effects of structural resonances on the production of shock-free high-amplitude pressure waves are discussed, especially concerning difficulties encountered when these resonances were in the frequency ranges of interest. Identifying features of a structural resonance are presented.

An Experimental/Numerical Study of Nonlinear Standing Waves in Resonators

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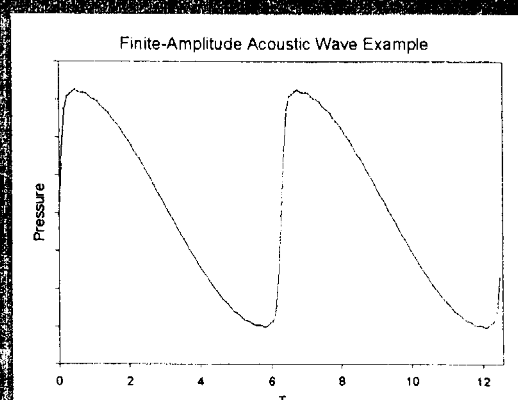
Overview

- Historical overview
- Nonlinearities in acoustics
- Experimental setup
- Numerical development
- Experimental and numerical results/comparison
- Conclusions

Historical Overview

- Finite-amplitude acoustic waves
- Experimental, numerical work
  - Experimentally find pressure limit
  - Reduce PDEs to ODEs assuming Fourier sawtooth waves
- Finite-amplitude exceeded
  - Lawrenson et. al. (1997), Ilinskii et. al. (1998)
  - Shaped resonators cause shaped pressure waves
  - Numerical model predicted waveforms well

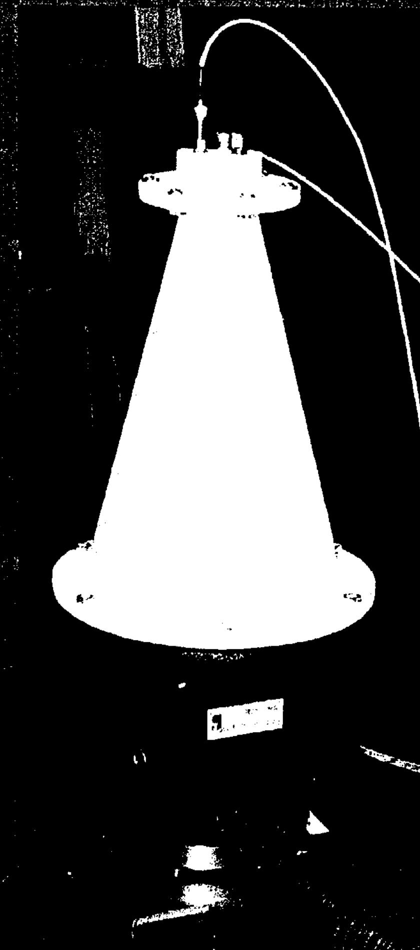


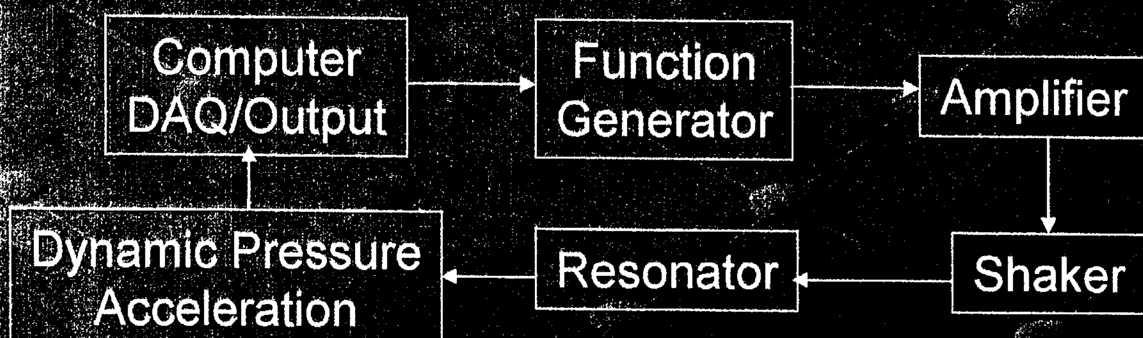
Objectives of Research

- Duplicate nonlinear acoustic effects in shaped cavities
- Modify existing numerical models
  - Account for shaped (cylindrical) center blockages in resonators
  - Extend model to minimize human input (automation)
- Directly compare results from model and experiments

Experimental Setup

- Small-scale (10lbs peak force) nonlinear acoustics test
- Cylinder and cone resonators; other designs in progress
- PID control easily implemented



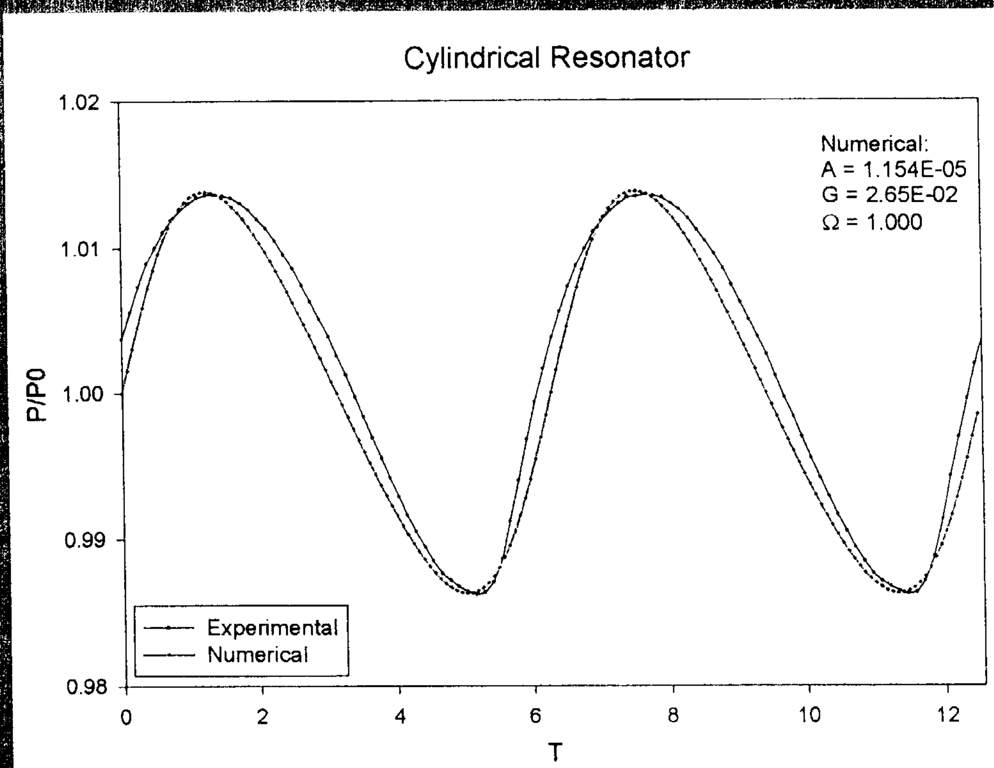


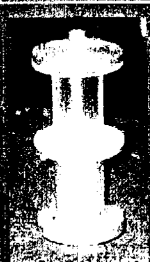
Numerical Development

- Navier-Stokes Equations
  - Mass, momentum, state
- Account for cavity shape and center blockage
  - Radius
  - X-derivative of radius
- Normalize equations
- Assume periodic solutions ($e^{i \omega_n t}$), transform to frequency domain
- Solve for velocity potential, velocity using multiple shooting method

Cylindrical Resonator

- Matched acceleration parameter
- Adjusted viscous dissipation (G)
- G larger than expected
  - Relative size of boundary layer
  - Small resonator, G difficult to determine

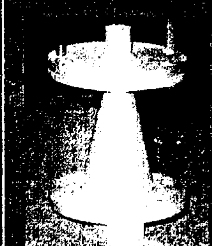


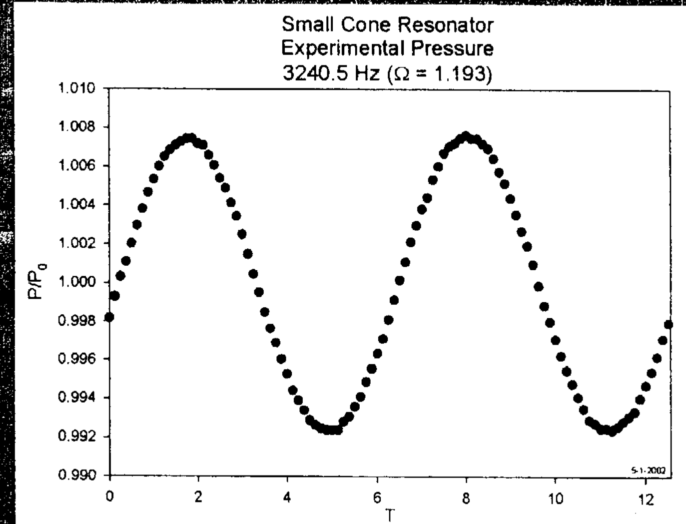


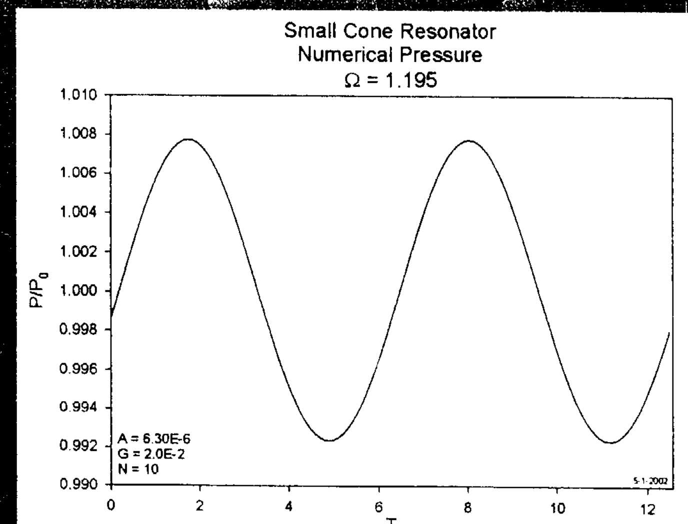
$r = 0.625''$, $0<x<5''$
($R = 0.125$, $0<X<1$)

Small Cone Resonator

- Approximated A and G terms
- Viscous dissipation term difficult to determine
- Good qualitative agreement
- Nonlinear effects present in small resonators







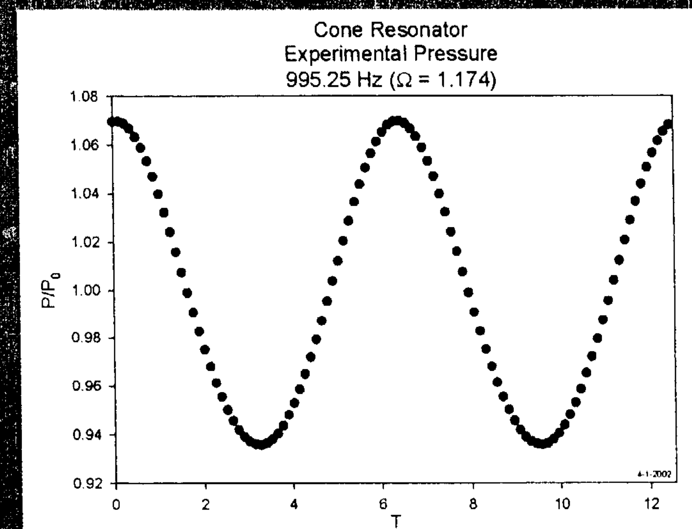
$r = 0.126'' + 0.202*x,\ 0<x<2.5''$

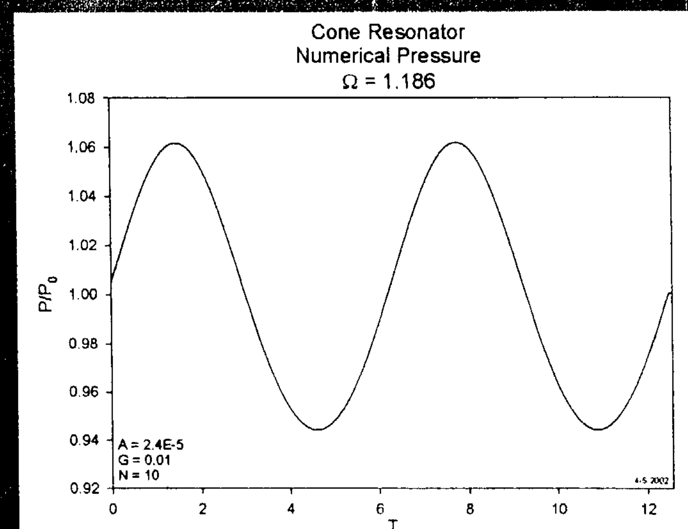
$(R = 0.0502+0.202*X,\ 0<X<1)$

Conical Resonator

- Acceleration waveform approximated
  - Resonator material not rigid
  - Top plate acceleration different than bottom plate
- Good qualitative agreement

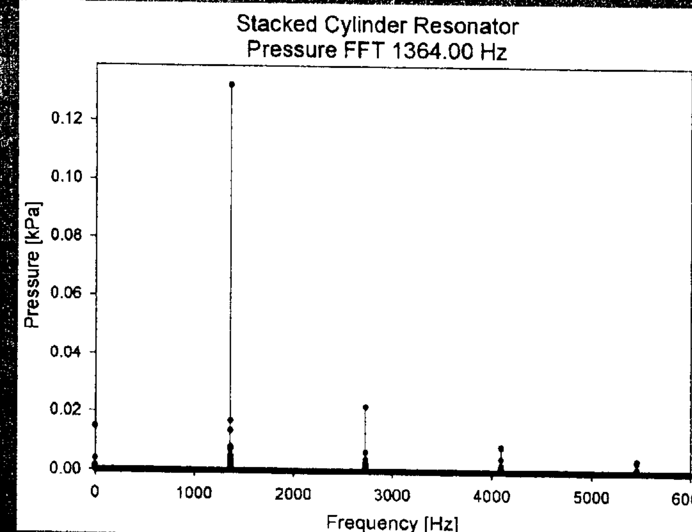
r = 0.523"+0.247*x, 0<x<8"
(R = 0.0652+0.247*X, 0<X<1)

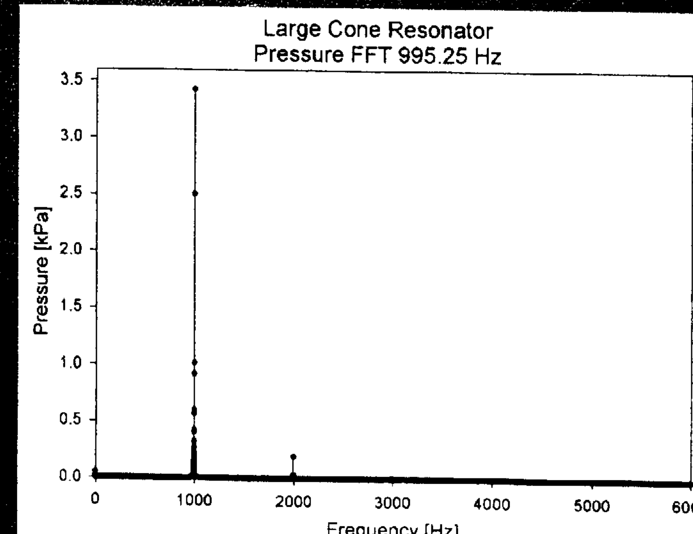




Experimental FFT comparison

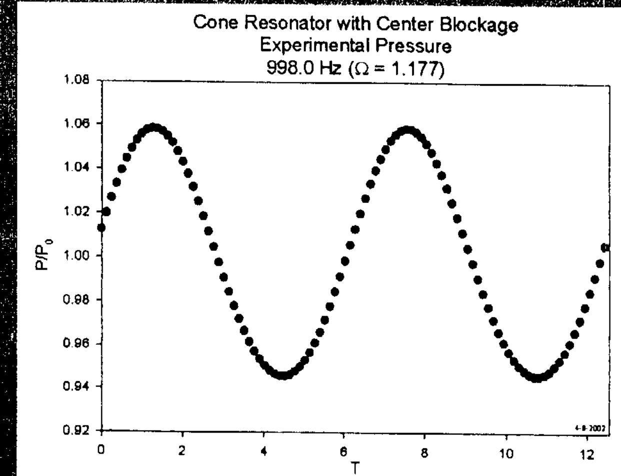
- Cylindrical Resonator
  - Several harmonics appear in FFT
  - Shows energy propagation between harmonics
- Conical Resonator
  - High harmonics do not appear
  - Energy confined to lower harmoncs

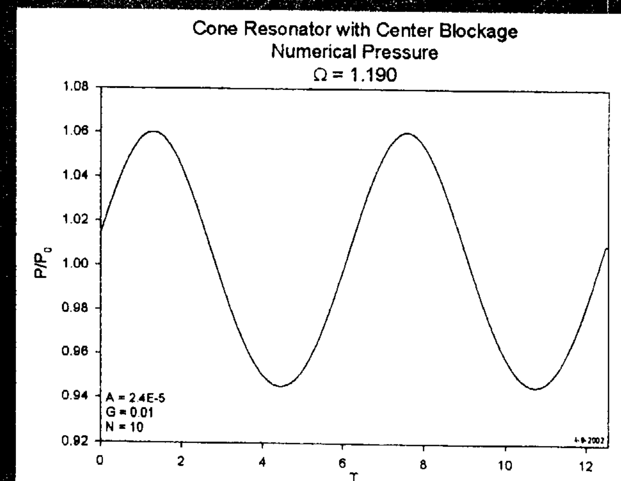




Cone with Center Blockage

- Acceleration approximated
- Both cases show higher frequencies
- High amplitude pressure waves present
- Strong qualitative agreement





$r = 0.523'' + 0.247*x,\ 0<x<8''$

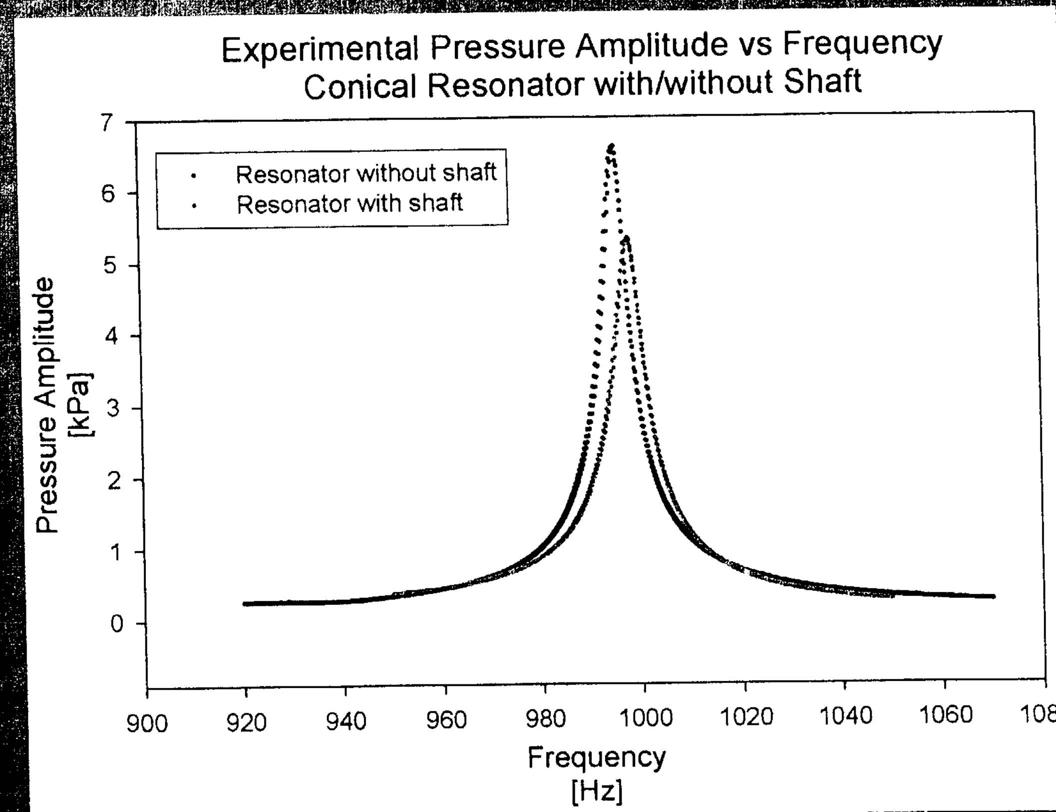
$(R = 0.0652 + 0.247*X,\ 0<X<1)$

$r_0 = 0.125,\ 0<x<8''$

$(R_0 = 0.0156,\ 0<X<1)$

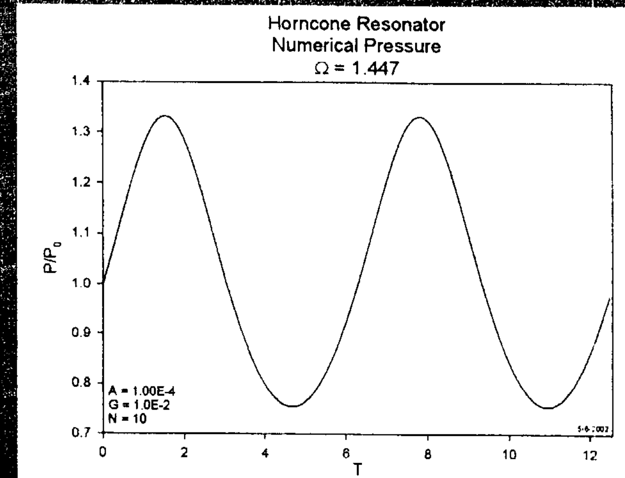
Conical Resonator Comparison

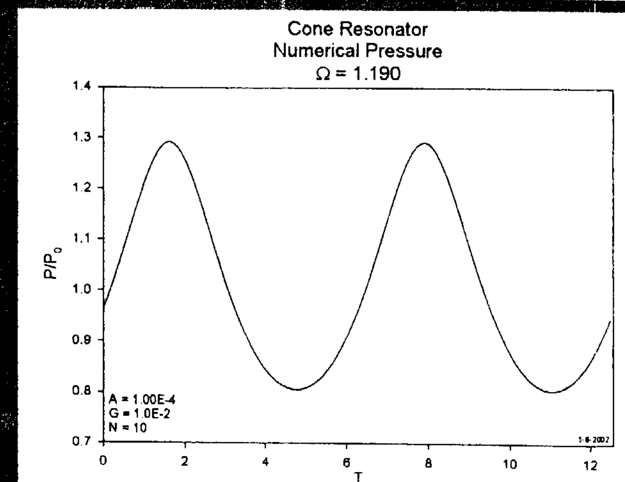
- Predicted values from code:
  - W = 1.186 without center blockage
  - W = 1.190 with center blockage
  - Ratio 0.9966
- Experiment:
  - 995.25 Hz without center blockage
  - 998.00 Hz with central blockage
  - Ratio 0.9972



Horncone Resonator

- Compare to cone numerical results at similar A
- Horncone predicted to have higher dynamic pressure
- Horncone resonant frequency higher





Cone: $R = 0.0652+0.247*X$, $0<X<1$

Horncone: $R = 0.02833*\cosh(6.62989*X)$, $0<X<0.25$

$R = 0.1622*\sinh(1.4316)*(X-0.25)$, $0.25<X<1$

What comes next?

- Change accelerometer location in cone
  - Direct comparison with model
  - Adjust model parameters
- Account for boundary layer effects
- Test new resonator designs
- Optimization routines

Concluding Remarks

- Shock-free pressure waves generated and verified
- Cylindrical center blockages do not hinder shock-free pressure waves
- Numerical model predicts acoustic behavior